Υποδείξεις : Φυλλάδιο 3

① α) $\sqrt{6\sqrt{12\sqrt{3\sqrt{9}}}} = \sqrt{6\sqrt{12\sqrt{3\cdot 3}}} = \sqrt{6\sqrt{12\cdot 3}} = \sqrt{6\cdot 6} = 6$

β) $\sqrt{86+2\sqrt{52-\sqrt{9}}} = \sqrt{86+2\sqrt{52-3}} = \sqrt{86+2\sqrt{49}} = \sqrt{86+2\cdot 7} = \sqrt{100} = 10$

γ) $\sqrt{\frac{2}{3}\sqrt{\frac{9}{2}\sqrt{\frac{1}{4}}}} = \sqrt{\frac{2}{3}\sqrt{\frac{9}{2}\cdot\frac{1}{2}}} = \sqrt{\frac{2}{3}\sqrt{\frac{9}{4}}} = \sqrt{\frac{2}{3}\cdot\frac{3}{2}} = \sqrt{1} = 1$

δ) $3\sqrt{18}+\sqrt{72}-\sqrt{50} = 3\sqrt{2\cdot 9}+\sqrt{2\cdot 36}-\sqrt{2\cdot 25} = 3\cdot 3\sqrt{2}+6\sqrt{2}-5\sqrt{2} = 10\sqrt{2}$

② $x>0$: $\sqrt{x}+\frac{1}{\sqrt{x}} \geq 2 \overset{\sqrt{x}>0}{\Leftrightarrow} x+1 \geq 2\sqrt{x} \Leftrightarrow (\sqrt{x})^2 - 2\sqrt{x}+1 \geq 0 \Leftrightarrow (\sqrt{x}-1)^2 \geq 0$

το οποίο ισχύει για κάθε $x>0$.

Ειδικότερα παίρνει τιμή το 0 που παίρνεται για $x=1$

③ $\frac{\alpha\sqrt{\alpha}-\beta\sqrt{\beta}}{\sqrt{\alpha}-\sqrt{\beta}} = \frac{(\alpha\sqrt{\alpha}-\beta\sqrt{\beta})(\sqrt{\alpha}+\sqrt{\beta})}{(\sqrt{\alpha}-\sqrt{\beta})(\sqrt{\alpha}+\sqrt{\beta})} = \frac{\alpha\sqrt{\alpha}^2-\beta\sqrt{\alpha}\sqrt{\beta}+\alpha\sqrt{\alpha}\sqrt{\beta}-\beta\sqrt{\beta}^2}{\sqrt{\alpha}^2-\sqrt{\beta}^2} =$

$= \frac{\alpha^2-\beta\sqrt{\alpha\beta}+\alpha\sqrt{\alpha\beta}-\beta^2}{\alpha-\beta} = \frac{(\alpha-\beta)(\alpha+\beta)}{\alpha-\beta} + \frac{\sqrt{\alpha\beta}(\alpha-\beta)}{\alpha-\beta} =$

$= \alpha+\beta+\sqrt{\alpha\beta}$

④ α) $|\sqrt{5}| = \sqrt{5}$ διότι $\sqrt{5}>0$, β) $|-\sqrt{3}| = \sqrt{3}$ διότι $-\sqrt{3}<0$

γ) $|1-\sqrt{2}| = \sqrt{2}-1$ (διότι $\sqrt{2}>1$) δ) $|\pi-3\sqrt{2}| = 3\sqrt{2}-\pi$ ($3\sqrt{2}>\pi$)

ε) $|\pi-\sqrt{7}| = \pi-\sqrt{7}$ ($\sqrt{7}<\sqrt{9}=3<\pi$) στ) $|2\sqrt{7}-6| = 2\sqrt{7}-6$ ($2<\sqrt{7}<3 \Rightarrow 2\sqrt{7}<6$)

ζ) $|2\pi-6| = 2\pi-6$ ($\pi>3 \Rightarrow 2\pi>6$) η) $|2\pi-6\sqrt{3}| = 6\sqrt{3}-2\pi$ ($\frac{3}{2}<\sqrt{3} \Rightarrow 9<6\sqrt{3}$)

θ) $|\sqrt{\pi-\sqrt{6}}| = \sqrt{\pi-\sqrt{6}}$ ($\sqrt{\pi-\sqrt{6}}>0$) ι) $|x^2-2xy+y^2| = |(x-y)^2| = (x-y)^2$

κ) $|\pi^2-6\pi+9| = |(\pi-3)^2| = (\pi-3)^2 = \pi^2-6\pi+9$

⑤ $\alpha<3<\beta$ α) $A = |\alpha-3|+|2\beta-6|-|\alpha-\beta| = |\alpha-3|+2|\beta-3|-|\alpha-\beta| =$

$\frac{\alpha-3<0}{\alpha-\beta<0}$ $\beta-3>0$ $-(\alpha-3)+2(\beta-3)+(\alpha-\beta) = -\alpha+3+2\beta-6+\alpha-\beta = \beta-3$

β) $B = |3-\alpha|+|\beta|-|\alpha-\beta| \overset{\beta>0}{\underset{\alpha-\beta<0}{3-\alpha>0}}$ $3-\alpha+\beta+\alpha-\beta = 3$

⑥ $|\alpha|<1$ τότε $\frac{|\alpha+1|-|\alpha-1|}{|\alpha+1|+|\alpha-1|} \overset{\alpha+1>0}{\underset{\alpha-1<0}{-1<\alpha<1}}$ $\frac{\alpha+1+\alpha-1}{\alpha+1-\alpha+1} = \frac{2\alpha}{2} = \alpha$

⑦ Αν $|x|\leq 3$, $|y|\leq 1$ τότε: $|3x-5y+2| \leq 16$, $|x^2-2y+1| \leq 12$

$|3x-5y+2| \leq |3x|+|5y|+2 = 3|x|+5|y|+2 \leq 3\cdot 3+5\cdot 1+2 = 16$

$|x^2-2y+1| \leq |x^2|+|2y|+1 = |x|^2+2|y|+1 \leq 9+2\cdot 1+1 = 12$

⑧ $\{0, \alpha+\beta, 2\} = \{1, 2, \alpha-\beta\}$

$0 = \alpha-\beta \Rightarrow \alpha=\beta$

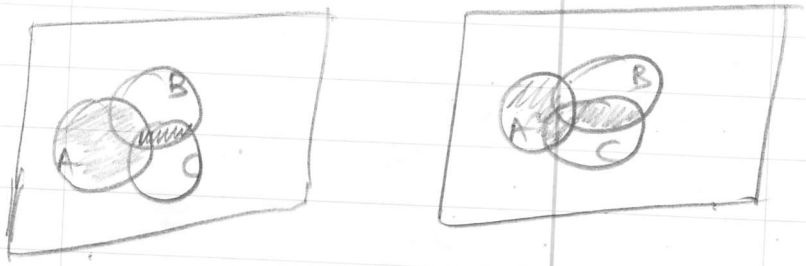
$\alpha+\beta=1 \Rightarrow 2\alpha=1 \Rightarrow \alpha=\frac{1}{2}$ $\quad \beta=\frac{1}{2}$

⑨ $A=\{x\in\mathbb{R}: 4x+3 \le 9+5x\}$ $\quad B=\{x\in\mathbb{R}: -8 \le x<2\}$

$A=\{x\in\mathbb{R}: 3-9 \le 5x-4x\} = \{x\in\mathbb{R}: -6 \le x\}$

$A\cap B=\{x\in\mathbb{R}: -6 \le x<2\} = [-6, 2)$

⑩ πχ α) $A\cup(B\cap C) \neq (A\cup B)\cap(A\cup C)$



⑪ $|x+y|+|y+z|+|z+x| = 2xyz(|x|+|y|+|z|) \Rightarrow xyz \le 1$

Αν $x=y=z=0$ ΟΚ Αν $0\neq$ τότε:

$$xyz = \frac{|x+y|+|y+z|+|z+x|}{2(|x|+|y|+|z|)} \le \frac{|x|+|y|+|y|+|z|+|z|+|x|}{2(|x|+|y|+|z|)} = 1$$