① $\frac{\alpha+\beta}{\beta} = \frac{\alpha}{\beta} + 1 = \frac{2}{3} + 1 = \frac{2}{3} + \frac{3}{3} = \frac{5}{3}$

② $\left[(x^2 \cdot y^{-3})^4 \cdot x^3 \cdot y^{18} : (x^{-2} \cdot y^3)^{-1}\right]^4 = \left[x^8 \cdot y^{-12} \cdot x^3 \cdot y^{18} \cdot x^{-2} \cdot y^3\right]^4 =$

$= (x^9 \cdot y^9)^4 = \left[(xy)^9\right]^4 = 1^{36} = 1$

③ $\left(\frac{-4x^{-2}y^2}{3xy^{-1}}\right)^2 \cdot \left(\frac{x^2y^{-3}}{2x^{-3}y^4}\right)^{-3} = \left(-\frac{4}{3}x^{-3}y^3\right)^2 \cdot \left(\frac{1}{2}x^5y^{-7}\right)^{-3} =$

$= \frac{16}{9}x^{-6}y^6 \cdot 8 \cdot x^{-15}y^{21} = \frac{2^7}{3^2}x^{-21}y^{27}$ $\frac{128}{9}$

④ $8 \cdot 4^2 \cdot 27 \cdot 9^3 = 2^3 \cdot (2^2)^2 \cdot 3^3 \cdot (3^2)^3 = 2^3 2^4 \cdot 3^3 \cdot 3^6 = 2^7 \cdot 3^9$

⑤ $(k-1)(k-2)(k-3) = 0 \Rightarrow k=1$ ή $k=2$ ή $k=3$

$(k+1)(k+2)(k+3) = \begin{cases} 1 \cdot 2 \cdot 3 \cdot 4 = 24 \\ 2 \cdot 3 \cdot 4 \cdot 5 = 60 \\ 3 \cdot 4 \cdot 5 \cdot 6 = 120 \end{cases}$

⑥

α) $\frac{\kappa\alpha+\kappa\beta}{\kappa\gamma+\kappa\delta} = \frac{\kappa(\alpha+\beta)}{\kappa(\gamma+\delta)} = \left(\frac{\alpha+\beta}{\gamma+\delta}\right)$ β) $\frac{\alpha\beta+\beta\gamma+\beta\delta}{\beta\epsilon} = \frac{\beta(\alpha+\gamma+\delta)}{\beta\epsilon} = \frac{\alpha+\gamma+\delta}{\epsilon}$

γ) $\frac{\kappa\alpha-\kappa\beta-\gamma\alpha+\gamma\beta}{\kappa\alpha+\kappa\beta-\gamma\alpha-\gamma\beta} = \frac{\kappa(\alpha-\beta)-\gamma(\alpha-\beta)}{\kappa(\alpha+\beta)-\gamma(\alpha+\beta)} = \frac{(\alpha-\beta)(\kappa-\gamma)}{(\alpha+\beta)(\kappa-\gamma)} = \frac{\alpha-\beta}{\alpha+\beta}$

δ) $\frac{2\kappa\alpha+2\kappa\beta+\lambda\alpha+\lambda\beta}{2\kappa\alpha-2\kappa\beta+\lambda\alpha-\lambda\beta} = \frac{2\kappa(\alpha+\beta)+\lambda(\alpha+\beta)}{2\kappa(\alpha-\beta)+\lambda(\alpha-\beta)} = \frac{\alpha+\beta}{\alpha-\beta}$

⑦ $(\alpha-\beta)\left(\frac{1}{\alpha}-\frac{1}{\beta}\right) = 0 \Rightarrow \alpha=\beta$: $(\alpha-\beta)=0 \Leftrightarrow \alpha=\beta$ ή $\frac{1}{\alpha}-\frac{1}{\beta}=0 \Leftrightarrow \beta-\alpha=0 \Rightarrow \beta=\alpha$

⑧ $\alpha<\beta$ και $\gamma<\delta \Rightarrow$ α) $2\alpha+3\gamma<2\beta+3\delta$ β) $2\alpha-5\delta<2\beta-5\gamma$ γ) $\frac{\alpha+\gamma}{2}+4<\frac{\beta+\delta}{2}+4$

$\begin{array}{l} 2\alpha<2\beta \\ 3\gamma<3\delta \\ \hline 2\alpha+3\gamma<2\beta+3\delta \end{array}$ | $\begin{array}{l} 2\alpha<2\beta \\ 5\gamma<5\delta \Rightarrow \\ -5\delta<-5\gamma \\ \hline 2\alpha-5\delta<2\beta-5\gamma \end{array}$ | $\begin{array}{l} \alpha<\beta \\ \gamma<\delta \\ \hline \alpha+\gamma<\beta+\delta \end{array} \Rightarrow \frac{\alpha+\gamma}{2} < \frac{\beta+\delta}{2} \Rightarrow \frac{\alpha+\gamma}{2}+4 < \frac{\beta+\delta}{2}+4$

⑨ $0<y<x \Rightarrow \frac{x-y}{x+y} ? \frac{x^2-y^2}{x^2+y^2}$ — Έστω $\frac{x-y}{x+y} < \frac{x^2-y^2}{x^2+y^2}$ ($x+y>0$, $x^2+y^2>0$) $\Leftrightarrow (x-y)(x^2+y^2) < (x^2-y^2)(x+y)$

$\Leftrightarrow (x-y)(x^2+y^2) < (x-y)(x+y)^2$ ($x-y>0$) $\Leftrightarrow x^2+y^2 < x^2+y^2+2xy \Leftrightarrow xy>0$ ισχύει ok

ή μ-διαφορά

⑩ Αν $\alpha,\beta,\gamma>0$ τότε: α) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} \geq 2$ β) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) \geq 8\alpha\beta\gamma$

γ) Αν επιπλέον $\alpha+\beta+\gamma=1 \Rightarrow \left(\frac{1}{\alpha}-1\right)\left(\frac{1}{\beta}-1\right)\left(\frac{1}{\gamma}-1\right) \geq 8$

α) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} \geq 2 \overset{\alpha\beta>0}{\Leftrightarrow} \alpha^2+\beta^2 \geq 2\alpha\beta \Leftrightarrow (\alpha-\beta)^2 \geq 0$ ισχύει

β) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) \geq 8\alpha\beta\gamma \Leftrightarrow (\alpha\beta+\alpha\gamma+\beta^2+\beta\gamma)(\gamma+\alpha) \geq 8\alpha\beta\gamma \Leftrightarrow \alpha\beta\gamma+\alpha\gamma^2+\beta\gamma^2+\gamma\beta^2+\alpha^2\beta+\alpha^2\gamma+\alpha\beta^2+\alpha\beta\gamma \geq 8\alpha\beta\gamma \Leftrightarrow$

$\alpha\gamma^2+\beta^2\gamma+\gamma^2\beta+\alpha^2\beta+\alpha^2\gamma+\alpha\beta^2+2\alpha\beta\gamma \geq 8\alpha\beta\gamma$

$\alpha(\beta^2+\gamma^2)+\gamma(\alpha^2+\beta^2)+\beta(\alpha^2+\gamma^2)+2\alpha\beta\gamma \geq 8\alpha\beta\gamma$

$\geq \alpha 2\beta\gamma + \gamma 2\alpha\beta + \beta 2\alpha\gamma + 2\alpha\beta\gamma = 8\alpha\beta\gamma$

$\left(\frac{1}{\alpha}-1\right)\left(\frac{1}{\beta}-1\right)\left(\frac{1}{\gamma}-1\right) = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\gamma}{\gamma}\right)$

$= \left(\frac{\beta+\gamma}{\alpha}\right)\left(\frac{\alpha+\gamma}{\beta}\right)\left(\frac{\alpha+\beta}{\gamma}\right) \overset{\beta}{\geq} \frac{8\alpha\beta\gamma}{\alpha\beta\gamma} = 8$