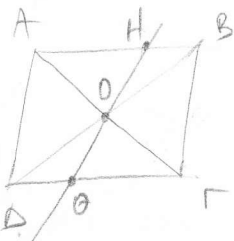


9

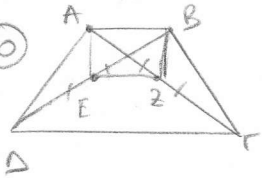


$$\begin{array}{l} \Delta \quad \begin{array}{l} AO = OG \\ DO = BO \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) OH = OG \\ \beta) AH = BG \end{array} \end{array}$$

$$\Delta \quad \begin{array}{l} \hat{AHO} = \hat{BOG} \\ AO = OG \\ \hat{BAH} = \hat{GBA} \\ \hat{AOH} = \hat{BOG} \end{array}$$

$\Gamma\Gamma\Gamma \Rightarrow OH = OG$
και
 $AH = BG$

10



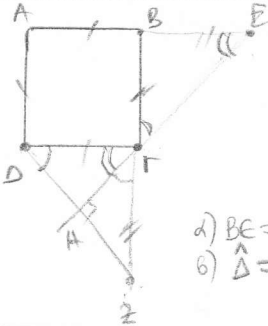
$$\begin{array}{l} \Delta \quad \begin{array}{l} AB \parallel \Gamma\Delta, AD = BG \\ \Gamma\Delta = 3AB \\ E, Z \text{ μέσο} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) EZ \parallel AB, EZ = AB \\ \beta) AE = ZB \text{ ορθ. τριγ.} \\ \gamma) AE, BZ \text{ ύψη} \end{array} \end{array}$$

$$\alpha) EZ = \frac{\Gamma\Delta - AB}{2} = \frac{3AB - AB}{2} = AB$$

$$\beta) AB = 1/2 EZ \text{ και } AZ = BE \text{ (γιατί?)}$$

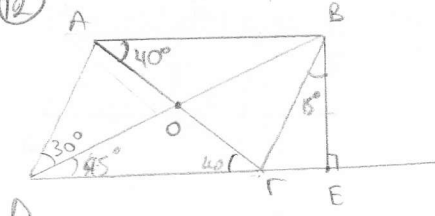
$$\gamma) AE \perp AD, BZ \perp AB$$

11



$$\begin{array}{l} \Delta \quad \begin{array}{l} AB \parallel \Gamma\Delta, BE = \Gamma Z \\ \beta) \hat{BEG} = \hat{\Delta Z\Gamma} \\ \gamma) \hat{H\Gamma Z} = \hat{\Gamma\Delta Z} \\ \delta) \hat{H\Gamma Z} \text{ ορθ.} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) BE = \Gamma Z, \Gamma D = BG, \hat{\Gamma} = \hat{B} = 90^\circ \\ \beta) \hat{\Delta} = \hat{B\Gamma E} = \hat{H\Gamma Z} \end{array} \end{array}$$

12



$$\begin{array}{l} \Delta \quad \begin{array}{l} BE \perp \Gamma\Delta, \hat{B\Gamma E} = 15^\circ \\ \hat{\Gamma\Delta B} = 40^\circ, \hat{BAA} = 30^\circ \end{array} \\ \hline \Sigma \quad \begin{array}{l} \hat{B\Gamma E}, \hat{B\Delta\Gamma}, \hat{A\Gamma\Delta}, \hat{\Delta} \end{array} \end{array}$$

$$\hat{B\Gamma E} = 90 - 15 = 75^\circ$$

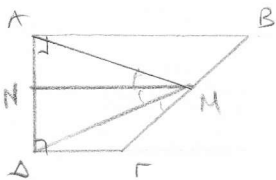
$$\hat{B\Delta\Gamma} = 15 + 90 = 105^\circ$$

$$\hat{A\Gamma\Delta} = 40^\circ \text{ εντός ευθείας} \dots$$

$$\hat{A\Delta\Gamma} = 75^\circ \text{ εντός ευθείας}$$

$$\hat{B\Delta\Gamma} = 45^\circ \quad \hat{\Delta} = 180 - 45 - 40 = 95^\circ$$

13



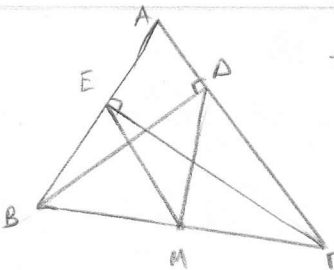
$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{A} = \hat{\Delta} = 90^\circ \text{ ΜΝ διαμέτρος} \\ \beta) \Gamma\Gamma = 2\Gamma\Delta \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) MA = MB \\ \beta) \hat{\Delta M\Gamma} = \hat{A\Delta N} \\ \gamma) \hat{A\Delta\Gamma} = 3\hat{B\Delta N} \end{array} \end{array}$$

α) Στο $\hat{A\Delta\Gamma}$ ΜΝ ύψος, διαμέτρος άρα $\hat{A\Delta N} = \hat{B\Delta N}$ και $MA = MB$

$$\beta) \hat{M\Gamma} = \frac{\beta\Gamma}{2} = \Gamma\Delta \Rightarrow \hat{\Delta M\Gamma} = \hat{\Gamma\Delta N} \left. \begin{array}{l} \hat{A\Delta N} = \hat{B\Delta N} \text{ εντός ευθείας} \\ \hat{A\Delta\Gamma} = \hat{B\Delta N} \end{array} \right\}$$

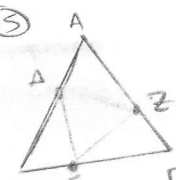
$$\gamma) \hat{A\Delta\Gamma} = 3\hat{B\Delta N} = 3\hat{B\Delta M}$$

14



$$\begin{array}{l} \Delta \quad \begin{array}{l} \Gamma E, BD \text{ ύψη} \\ \beta) EM = MA \end{array} \\ \hline \Sigma \quad \begin{array}{l} \hat{B\Gamma\Delta} \text{ ορθογώνιο} \\ \hat{\Delta M} \text{ διαμέτρος υψος} \dots \\ \hat{\Delta M} = \frac{\beta\Gamma}{2} \\ \hat{B\Gamma E} \text{ ορθογώνιο} \dots \\ \hat{EM} = \frac{\beta\Gamma}{2} \end{array} \end{array}$$

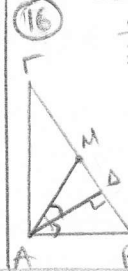
15



$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{BDE} = \hat{AAZ} = \\ = \hat{\Gamma EZ} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \hat{AD} = \hat{BE} = \hat{\Gamma Z} \\ \hat{AE} = \hat{AF} = \hat{BF} \end{array} \end{array}$$

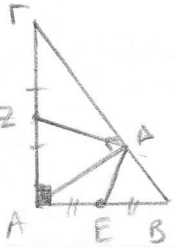
$$\alpha) \hat{DE} = \hat{EZ} = \hat{\Delta Z}$$

16



$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{A} = 90^\circ \text{ ΜΝ διαμ. ΑΔ ύψη} \\ \beta) \hat{M\Delta B} = \hat{B} - \hat{\Gamma} \\ \hat{\Delta A B} = \hat{\Gamma} \left\{ \begin{array}{l} \hat{\Delta A} \perp \hat{B\Gamma} \\ \hat{A B} \perp \hat{\Gamma A} \end{array} \right. \\ \hat{M\Delta B} = \hat{B} \Rightarrow \\ \hat{M\Delta A} + \hat{\Delta A B} = \hat{B} \Rightarrow \\ \hat{M\Delta A} = \hat{B} - \hat{\Gamma} \end{array} \end{array}$$

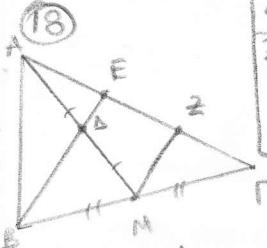
17



$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{A} = 90^\circ, EZ \text{ μέσο ΑΔ ύψη} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) \hat{Z\Delta A} = \hat{Z\Delta D} \\ \beta) \hat{C\Delta A} = \hat{C\Delta D} \\ \gamma) \hat{Z\Delta E} = 90^\circ \\ \delta) \text{ αν } AB = \beta\Gamma = \Delta EZ \\ \text{ορθογώνιο + 100\%} \end{array} \end{array}$$

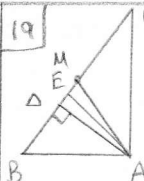
- α) $\hat{\Gamma\Delta A}$ ορθογώνιο, ΔZ διαμέτρος υποτενύσας $\Rightarrow Z\Delta = Z\Lambda$
- β) $\hat{A\hat{B}\Delta}$ ορθογώνιο, ΔE διαμέτρος...
- γ) $\hat{Z\Delta E} = \hat{Z\Delta E} \dots$
- δ) $Z\Delta = \frac{\beta\Gamma}{2} = \frac{AB}{2} = \Delta E \dots$

18



$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{M}, \hat{D} \text{ μέσο} \\ \hat{M Z} \parallel \hat{B E} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) AE = EZ \\ \beta) EZ = Z\Gamma \\ \gamma) AE = \frac{\beta\Gamma}{2} \end{array} \end{array}$$

19

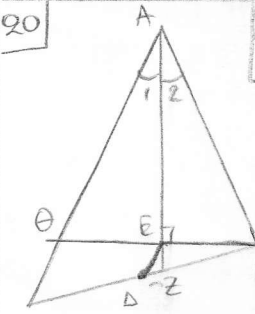


$$\begin{array}{l} \Delta \quad \begin{array}{l} \text{ΑΔ ύψη, ΑΕ διαμ, ΑΜ διαμ } \hat{A} = 90^\circ \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) \hat{M\Delta\Gamma} = \hat{\Gamma} \\ \beta) \hat{\Delta\Delta B} = \hat{\Gamma} \\ \gamma) \text{ ΑΕ διαμ } \hat{M\Delta D} \end{array} \end{array}$$

$$MA = \beta\Gamma/2 \Rightarrow \dots \alpha), \beta)$$

γ) ΔΕΕ αβκλ non 16

20



$$\begin{array}{l} \Delta \quad \begin{array}{l} \hat{A B C \Gamma} \hat{A Z} \text{ διαμ } BE \perp AZ, \hat{\Delta} \text{ μέσο} \end{array} \\ \hline \Sigma \quad \begin{array}{l} \alpha) AB = \beta\Gamma \quad \beta) \hat{\Delta E} \parallel \hat{\Gamma} \quad \gamma) \hat{\Delta E} = \frac{\beta\Gamma}{2} \quad \delta) \hat{\Delta E} = \frac{\beta\Gamma - \beta\Delta}{2} \end{array} \end{array}$$

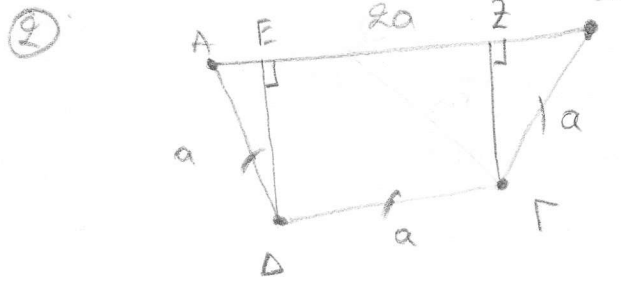
α) Στο τρίγωνο $\hat{A\beta\theta}$ $AE \perp \beta\theta$, $\hat{A}_1 = \hat{A}_2 \Rightarrow \hat{A\beta\theta}$ ισοσκελές

β) $E, \hat{\Delta}$ μέσο $EM \parallel \beta\theta \hat{\Gamma} \Rightarrow \dots$

$$\delta) \hat{\Delta E} = \frac{\beta\Gamma}{2} = \frac{\beta\Gamma - \beta\Delta}{2} = \frac{\beta\Gamma - \beta\Delta}{2}$$

- α) Στο $\hat{A M Z}$ $\hat{\Delta}$ μέσο $AE \parallel MZ \Rightarrow E$ μέσο $AZ \Rightarrow AE = EZ$
- β) Στο $\hat{\Gamma B E}$ M μέσο, $MZ \parallel BE \Rightarrow Z$ μέσο $\Gamma E \Rightarrow EZ = Z\Gamma$
- γ) $EF = EZ + Z\Gamma = 2AE \Rightarrow AE = \frac{EF}{2}$

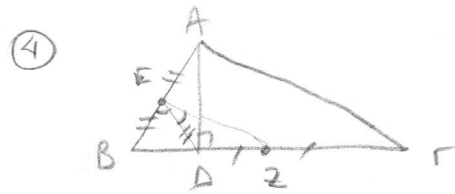
① $w = ?$ $180 - w = 3(90 - w) \Leftrightarrow 180 - w = 270 - 3w$
 $\Leftrightarrow 2w = 90 \Leftrightarrow w = 45^\circ$



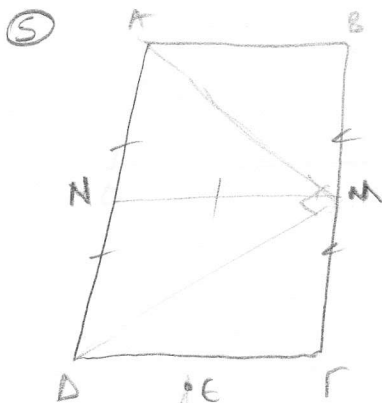
Δ $AB = 2a$ $BF = FD = DA = a$
 Σ a) $AE = 2B = \frac{a}{2}$
 b) $\hat{A} = \hat{B} = 60^\circ$

$\Delta AED = \Delta FDB$ unot. + xod.
 $\Gamma \Delta \Sigma$ $\text{opd. } \# \Rightarrow \epsilon \Sigma = a \Rightarrow AE + 2B = a$
 $AF = 2B$
 $\Delta \hat{A} \hat{B} \hat{E}$ $\text{opd. } AE = \frac{1}{2} AB \Rightarrow \hat{A} = 30^\circ \Rightarrow \hat{A} = \hat{B} = 60^\circ$

③ $R + r = \frac{P}{2} + \frac{3P}{2} = \frac{4P}{2} = 2P$ $R - r = \frac{3P}{2} - \frac{P}{2} = P$

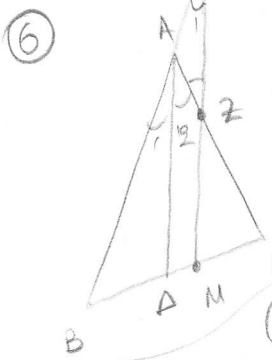


Δ $\hat{B} = \hat{\Gamma} + 30^\circ$ ϵ, Σ $\mu\epsilon\sigma\alpha$ $2\hat{\Gamma} = 2\hat{B} - 60$
 Σ a) $\Delta \hat{E} \hat{B} = 120^\circ - 2\hat{\Gamma}$ $\Leftrightarrow \Delta \hat{C} \hat{B} = 120 - 2\hat{B} + 60 = 180 - 2\hat{B}$
 b) $2\hat{E} \hat{B} = 150^\circ - 2\hat{\Gamma}$ $\text{nou igxuei stou gw } \Delta \hat{A} \hat{B} \hat{A} \hat{A} = 30^\circ$
 c) $2\hat{E} \hat{A} = ?$ $\text{opa } \epsilon \Delta = \epsilon B = \epsilon A.$
 e) $2\hat{E} \hat{B} = \hat{A} = 180 - \hat{B} - \hat{\Gamma} = 180 - \hat{\Gamma} - 30^\circ - \hat{\Gamma} = 150^\circ - 2\hat{\Gamma}$
 o) $2\hat{E} \hat{A} = 2\hat{E} \hat{B} - \Delta \hat{C} \hat{B} = 150 - 2\hat{\Gamma} - 180 + 2\hat{\Gamma} = 30^\circ$



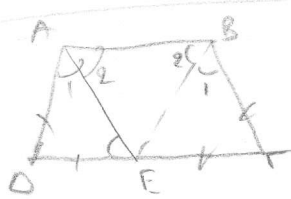
Δ	$AB + \Gamma D = AD$ $UN \mu\epsilon\sigma\alpha$
Σ	i) $MN = \frac{AD}{2}$ ii) $\hat{M} \hat{A} \hat{D} = 90^\circ$ iii) $AM \delta \times \hat{A}$ $DN \delta \times \hat{D}$

i) $MN = \frac{AB + \Gamma D}{2} = \frac{AD}{2}$
 ii) $MN = AN = DN \Rightarrow \dots \Rightarrow \hat{M} = 90^\circ$



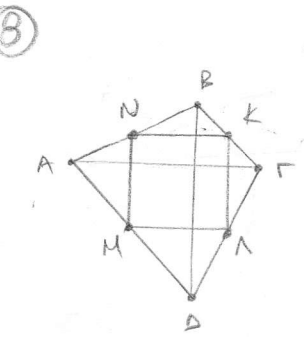
Δ	AD $\delta \times \alpha \mu \sigma \alpha$ $HE \parallel AD$
Σ	i) $\epsilon \hat{A} \hat{Z}$ isoxelias ii) $BE + \Gamma Z = AB + \Gamma F$

i) $\hat{A}_1 = \hat{E}$
 $\hat{A}_2 = \hat{Z}$
 $\hat{A}_1 = \hat{A}_2$
 ii) $BE + \Gamma Z = BA + AE + \Gamma Z = BA + AZ + \Gamma Z = BA + \Gamma F$



Δ	AE $\delta \times \alpha \mu \sigma \alpha$ $\Delta \Gamma = AD + BF$
Σ	i) $\Delta \epsilon = \Delta A$ ii) $\Gamma \epsilon = \Gamma B$ iii) BE $\delta \times \alpha \mu \sigma \alpha$

i) $AB \parallel \Gamma D \Rightarrow \hat{E} = \hat{A}_2$
 $\hat{A}_1 = \hat{A}_2 \Rightarrow \hat{A}_1 = \hat{E}$
 iii) $\Delta \Gamma = AD + BF$
 $\Delta \epsilon + \Gamma \epsilon = AD + BF \Rightarrow \Gamma \epsilon = BF$
 iii) $\hat{E} = \hat{B}_1, \hat{E} = \hat{B}_2 \Rightarrow \hat{B}_1 = \hat{B}_2$



Δ	$AF = BA$ $AF \perp BA$ M, N, K, A $\mu\epsilon\sigma\alpha$
Σ	i) $MN \parallel KA, MN = KA$ ii) $MN \perp MA$ iii) $MNKA$ tuplogwos

i) $\Delta \hat{A} \hat{B} \hat{D}$ UN $\text{enwvaw } \mu\epsilon\sigma\alpha$ strophw \dots
 ii) (i) + $AF \perp BA$
 iii) (i) + (ii)